Bose-Einstein Correlations for Expanding Finite Systems

or

From a Hot Fireball to a Snow-Flurry

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Abstract

Most boson emitting sources contain a core of finite dimensions surrounded by a large halo, due to long-lived resonances like ω, η, η', K^0 etc. When the Bose-Einstein correlation (BEC) function of the core can be determined we show that its intercept (λ) measures, as a function of momentum, the square of the fraction of core particles produced. A simultaneos measurement of BEC and the single-particle distributions can thus determine the characteristics of the core.

There are two types of scales present in the space-like as well as in the time-like directions in a model-class describing a cylindrically symmetric, finite, three-dimensionally expanding boson source. One type of the scales is related to the finite lifetime and geometrical size of the system, the other type is

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governed by the change of the local momentum distribution in temporal and spatial direction. If the geometrical sizes of the core are sufficiently large the parameters of the BEC function obey the m_t-scaling observed in SPb and $PbPb$ reactions at CERN. The model can describe the measurements of the single- and two-particle distributions in the central region of SPb reactions. A fit to experimental data shows that the freeze-out of hadrons occurs at a larger volume and at a much lower temperature than that given by the measurement of the inverse slope of the m_t-spectrum and standard BEC analysis.

I. INTRODUCTION.

Hanbury-Twiss correlations were discovered 40 years ago, revealing information about the angular diameters of distant stars [1]. The method, also referred to as intensity interferometry, extracts information from the quantum statistical correlation function for (partially) chaotic fields. The method has extensively been applied to the study of the freeze-out geometry in high energy nucleus-nucleus collisions as well as in elementary particle reactions. For recent reviews see refs. [2,3].

Accumulating evidence indicates that the space-time structure of the pion emission in heavy ion reactions at the 200 AGeV bombarding energy region at CERN SPS has a peculiar feature, namely that the boson emission can be approximately divided into two parts: the centre and the halo [4–7]. The central part corresponds to a direct production mechanism e.g. hydrodynamical evolution or particle production from excited strings, followed by subsequent rescattering of the particles, while the surrounding halo corresponds to pions emitted from the decay of long-lived hadronic resonances, like ω, η, η' and K^0, which have a mean decay length of more than 20 fm. These long-lived resonances will decay and give rise to mainly low momentum pions. However, the momentum distribution of the emitted pions is independent from the position of the decay which in turn is smeared out over a large region due to the long mean decay length. This mechanism will result in pions of similar (low) momentum

from a very large volume. This is in contrast to the core which when expanding in a hydrodynamical manner will have pions of similar momentum from a relatively small volume. We will investigate the phenomenological consequences of such a structure where there is a characteristic length-scale of the core which is smaller than any length-scale that can be connected to the surrounding halo. In the case of halo length-scales larger than 20 fm they give rise to a sharp peak of the correlation function in the $Q \leq \hbar/R \approx 10$ MeV/c region. This region is strongly influenced by the Coulomb interaction so the accuracy of the correlation function determination will be dependent on proper Coulomb corrections which constitute a problem in itself. Furthermore, this $Q < 10$ MeV/c region is very difficult to measure due to that we must properly determine two very close tracks which make experimental systematic errors largest in this region of the correlation function even for high resolution experiments like the NA35 and NA44 high energy heavy ion experiments at CERN.

We will show that even if we cannot measure reliably the correlation function below $Q < 10$ MeV/c important new information about the particle production in a core/halo scenario can be given by analyzing both the invariant momentum distribution of single particles and the correlation function for $Q > 10$ MeV/c. We will apply our approach to NA44 data on SPb reactions at 200 AGeV/c.

We investigate the core/halo scenario utilizing analytical results in the Wigner-function formalism. This formalism is well suited to describe both the invariant momentum distribution (IMD) and the Bose-Einstein correlation function (BECF). We utilize the version of the formalism discussed in refs. [8,3] and applied to analytic calculations in refs. [10–15] recently.

II. THE WIGNER-FUNCTION FORMALISM

In the Wigner-function formalism, the one-boson emission is characterized by the emission function, $S(x,p)$, which is the time derivative of the single-particle Wigner function. Here $x = (t, \vec{r}\,) = (t, r_x, r_y, r_z)$ denotes the four-vector in space-time, and

$p = (E, \vec{p}) = (E, p_x, p_y, p_z)$ stands for the four-momentum of the on-shell particles with mass $m = \sqrt{E^2 - \vec{p}^2}$. The transverse mass is defined as $m_t = \sqrt{m^2 + p_x^2 + p_y^2}$ and $y = 0.5 \log \left(\dfrac{E + p_z}{E - p_z} \right)$ stands for the rapidity. Attention is drawn to the point that the normal size character x stands for a four-vector, normalsize y is the symbol for the rapidity while the subscripts $_x$ and $_y$ index directions in coordinate and momentum space.

For chaotic sources the two-particle Wigner-function can be expressed in terms of the symmetrized products of the single-particle Wigner functions. Based on this property, the two-particle Bose-Einstein correlation functions can be determined from the single-particle emission function solely. These relations are especially simple in terms of the Fourier-transformed emission function,

$$\tilde{S}(\Delta k, K) = \int d^4x \; S(x, K) \; \exp(i \Delta k \cdot x), \tag{1}$$

where

$$\Delta k = p_1 - p_2, \qquad\qquad K = \frac{p_1 + p_2}{2} \tag{2}$$

and $\Delta k \cdot x$ stands for the inner product of the four-vectors. The invariant momentum distribution of the emitted particles is given by

$$\frac{d^2 n}{dy \, dm_t^2} = \tilde{S}(\Delta k = 0, K = p). \tag{3}$$

The IMD is normalized to unity as

$$\int dy \, dm_t^2 \frac{d^2 n}{dy \, dm_t^2} = 1. \tag{4}$$

The two-particle BECF-s are prescribed in terms of our auxiliary function,

$$C(K, \Delta k) \simeq 1 + \frac{\mid \tilde{S}(\Delta k, K) \mid^2}{\mid \tilde{S}(0, K) \mid^2}, \tag{5}$$

as was presented e.g. in ref. [8,3]. The effect of final state Coulomb and Yukawa interactions shall be neglected as implicitly assumed by the above equation, and we assume completely chaotic emission.

The Wigner-functions are in general complex valued functions, being the quantum analogue of the classical phase-space distribution functions. For the purpose of Monte-Carlo simulations of the Bose-Einstein correlation functions the off-shell Wigner-functions are approximated by the on-shell classical emission functions from the simulation [8,9]; in case of analytic calculations they are usually modeled by some positive valued functions [10–15]. Neither of these approximations seems to be necessary: in fact we need only simplifying assumptions as listed below.

III. THE CORE/HALO MODEL

Assumption 0. There is a resolvable length-scale in the core

Assumption 1. The bosons are emitted either from a central *core* or from the surrounding *halo*. The respective emission functions are indicated by $S_c(x,p)$ and $S_h(x,p)$, and f_c indicates the fraction of the bosons emitted from the central part. According to this assumption, the complete emission function can be written as

$$S(x,p) = f_c S_c(x,p) + (1 - f_c) S_h(x,p). \tag{6}$$

Both the emission function of the core and that of the halo are normalized similarly to the complete emission function, normalized by eqs. (1,3,4).

Assumption 2. We assume that the emission function which characterizes the halo changes on a scale R_H which is larger than $R_{max} \approx \hbar/Q_{min}$, the maximum length-scale resolvable by the intensity interferometry microscope. Q_{min} is taken as 10 MeV/c in our considerations. However, the smaller central part of size R_c is assumed to be resolvable,

$$R_H > R_{max} > R_c. \tag{7}$$

This inequality is assumed to be satisfied by all characteristic scales in the halo and in the central part, e.g. in case the side, out or longitudinal components [16,10] of the correlation function are not identical.

Based on *Assumption 1* the IMD can be re-written as

$$\frac{d^2 n}{dy\, dm_t^2} = f_c \frac{d^2 n_c}{dy\, dm_t^2} + (1 - f_c) \frac{d^2 n_h}{dy\, dm_t^2}, \tag{8}$$

where the subscripts c, h index the contributions by the central and the halo parts to the IMD given respectively by

$$\frac{d^2 n_i}{dy\, dm_t^2} = \tilde{S}_i(\Delta k = 0, K = p) \tag{9}$$

for $i = c, h$. The normalization condition according to *Assumption 1* reads as

$$\int dy\, dm_t^2 \frac{d^2 n_i}{dy\, dm_t^2} = 1 \tag{10}$$

for $i = c, h$. Note that the IMD as expressed by eq. (8) includes the possibility that the IMD for the bosons of the halo is different from the bosons emerging from the central part. Thus the relative contribution of the halo and the core is a function of the momentum in this model.

The BECF is expressed by

$$C(K, \Delta k) = 1 + \frac{\mid f_c\, \tilde{S}_c(\Delta k, K) + (1 - f_c)\, \tilde{S}_h(\Delta k, K) \mid^2}{\mid f_c\, \tilde{S}_c(\Delta k = 0, K = p) + (1 - f_c)\, \tilde{S}_h(\Delta k = 0, K = p) \mid^2}, \tag{11}$$

which includes interference terms for boson pairs of (c, c) (c, h) and (h, h) type. Due to the assumption that the emission is completely chaotic, the exact value of the BECF at the $\Delta k = 0$ is always 2 in this model.

The measured two-particle BECF is determined for $Q > Q_{min} \approx 10$ MeV/c, and any structure within the $\Delta k < Q_{min}$ region cannot be resolved. However, the (c, h) and (h, h) type boson pairs create a narrow peak in the BECF exactly in this Δk region according to

eq. (11), which cannot be resolved due to *Assumption 2*. According to this assumption, the Fourier-transformed emission function of the halo for non-zero relative momenta vanishes at the given resolution Q_{min}. At zero relative momentum the same quantity gives the single particle momentum distribution from the halo, which is not effected by the *two*-particle resolution.

Including the finite resolution effect, symbolized by the horizontal bar, the measured BECF can be written as

$$\overline{C(\Delta k, K)} = 1 + \lambda_*(K = p; Q_{min}) \frac{\mid \tilde{S}_c(\Delta k, K) \mid^2}{\mid \tilde{S}_c(\Delta k = 0, K = p) \mid^2}, \tag{12}$$

where

$$\lambda_*(K = p; Q_{min}) = \frac{\mid f_c \tilde{S}_c(\Delta k = 0, K = p) \mid^2}{\mid f_c \, \tilde{S}_c(\Delta k = 0, K = p) + (1 - f_c) \, \tilde{S}_h(\Delta k = 0, K = p) \mid^2}, \tag{13}$$

or expressed with the IMDs

$$\lambda_*(p; Q_{min}) = \frac{\mid f_c \dfrac{d^2 n_c}{dy \, dm_t^2} \mid^2}{\mid \dfrac{d^2 n}{dy \, dm_t^2} \mid^2}. \tag{14}$$

The IMD is connected to the cross-sections by the relations

$$\frac{1}{\sigma} \frac{d^2 \sigma_i}{dy \, dm_t^2} = < n_i > \frac{d^2 n_i}{dy \, dm_t^2} \tag{15}$$

for $i = c, h$. From Eq. (8) we deduce $f_c = \dfrac{< n_c >}{< n >}$. The effective intercept $\lambda_*(p; Q_{min})$ is then also given by

$$\lambda_*(p; Q_{min}) = \frac{\mid \dfrac{d^2 \sigma_c}{dy \, dm_t^2} \mid^2}{\mid \dfrac{d^2 \sigma}{dy \, dm_t^2} \mid^2}. \tag{16}$$

The effective intercept parameter λ_* shall in general depend on the mean momentum of the observed boson pair, and we have $0 \leq \lambda_* \leq 1$. The equation above shows that this *effective* λ_* *has a very simple interpretation as the square of the fraction of core particles to all particles emitted*. We also see that this equation makes it possible to measure the cross-section of the bosons produced in the central core with the help of the combined use of the BECF parameter $\lambda_*(y, m_t)$ and the cross-section of all particles.

Note that the $\lambda_*(y, m_t)$ intercept parameter in our picture reflects the drop in the BECF from its exact value of 2 due to the contribution from the resonance halo, when measured with the finite two-particle momentum resolution Q_{min}. Thus the measured intercept, $\lambda_*(y, m_t)$ does *not* coincide with the exact value of the BECF at zero relative momentum within this description.

The effect of the halo is to introduce a momentum-dependent effective intercept parameter to the BECF. The shape of the BECF for $Q > Q_{min}$ shall be solely determined by the freeze-out phase-space distribution in the central part. This central part or core is usually well accessible to hydrodynamical calculations.

Note also that this result has in principle nothing to do with the similar relation obtained for the case of partially coherent, partially chaotic fields, where $\lambda_* = f_{inc}^2$, and $f_{inc} =< n_{inc} > / < n_{tot} >$ gives the fraction of the total multiplicity in the chaotic field, although the relation formally is similar. For the case of partially coherent fields, the Bose-Einstein correlation function contains an interference term in between the coherent and the chaotic fields which leads to a double-Gaussian or double-exponential structure [17], emphasized recently in ref. [18]. In our case, the BECF contains not three but only two terms, and it looks very much like the BECF would look like for the case the halo were missing i.e. $f_c = 1$.

Note also, that the focus is *not* on the reduction of the intercept of the BECF due to the halo (resonance decays) but more precisely on the *momentum-dependence* of this reduction. The fact of the reduction has been known before [5,4,8,9], however its momentum dependence was not utilized as far as we know. Within our formalism however it is straightforward to show that the core fraction of bosons can be estimated by

$$f_c = \int_{-\infty}^{\infty} dy \int_{m^2}^{\infty} dm_t^2 \sqrt{\lambda(y, m_t)} \frac{d^2 n}{dy\, dm_t^2} \tag{17}$$

and we also have

$$\min \sqrt{\lambda(y, m_t)} \, \leq \, f_c \, \leq \, \max \sqrt{\lambda(y, m_t)}. \tag{18}$$

Application. Present NA44 data [19] for central $S + Pb$ reactions at CERN SPS with 200 AGeV bombarding energy show an approximately m_t independent intercept parameter: $\lambda_{\pi^+} = 0.56 \pm 0.02$ and 0.55 ± 0.02 at the quite different mean transverse momenta of 150 MeV and 450 MeV, respectively. This suggests that in the midrapidity region the momentum distribution of the resonance halo is similar to that of the central part, and the halo contains $1 - \sqrt{\lambda} = 25 \pm 2$ % of all the pions. For this special case of $\lambda = f_c^2 = const$ we have the following simplified equations for the IMD and the BECF:

$$\frac{d^2 n}{dy\, dm_t^2} = \frac{d^2 n_c}{dy\, dm_t^2} = \frac{d^2 n_h}{dy\, dm_t^2}, \tag{19}$$

$$\overline{C(\Delta k, K)} \, = \, 1 + f_c^2 R_c(\Delta k, K). \tag{20}$$

I.e. the only apparent effect of the halo is to reduce the intercept parameter of the measured BECF to $\lambda = f_c^2$ while the IMD and the $(\Delta k, K)$ dependence of the BECF is determined by the central part *exclusively*.

Pions coming from the resonance-halo surrounding a strongly interacting centre are predominant at lower values of the transverse momentum according to the SPACER calculations in ref. [10] and RQMD calculations in ref. [9], similarly to the results of calculations with HYLANDER [5] and the hydrodynamical calculations of the Regensburg group [20]. The resonance fraction as a function of the transverse momentum was explicitly shown in the publications [9,5]. Each of the mentioned models predicts that the halo of resonances produces predominantly low momentum pions, and they predict that the effective intercept $\lambda_*(p; Q_{min})$ should increase with increasing transverse mass in contrast to the measured values.

The constancy of the effective parameter $\lambda_*(y, m_t)$ with respect to m_t suggests a mechanism of enhancing the low momentum particles in the core. Such an enhancement has a natural explanation in a hydrodynamical description of an expanding core as we will present below, see also [21,13]. The mechanism is a simple volume effect, as pointed out in [15]. To calculate the cross-section you integrate over the volume from where the particles of a given momentum are emitted. This volume is measured by the BECF. In the NA44 experiment all three radii of this volume are equal and they are inversely proportional the the square root of m_t [19]. This will give an extra volume factor, $V_* \propto (m_t)^{-3/2}$, enhancing the single particle cross-section at low m_t. We will return to this point.

Discussion. The general result for the BECF of systems with large halo coincides with the most frequently applied phenomenological parameterizations of the BECF in high energy heavy ion as well as in high energy particle reactions [2]. Previously, this form has received a lot of criticism from the theoretical side, claiming that it is in disagreement with quantum statistics [18] or that the λ parameter is just a kind of fudge parameter or a measure of our ignorance, which has been introduced to make theoretical predictions comparable to data. Now one can see that this type of parameterizations can be derived with a standard inclusion of quantum statistical effects – if we assume that we discuss interferometry for systems with large halo. The fact that most of the studied reactions, including $e^+ + e^-$ annihilations, various lepton-hadron and hadron-hadron reactions, nucleon-nucleus and nucleus - nucleus collisions are phenomenologically well describable [2] with variations of eq. (12) thus does not exclude the possibility that these high energy reactions all create boson emitting systems which include a large halo.

The applicability of the halo picture to a given reaction is not necessarily the only one possible explanation of a reduced intercept. It is known that the final state interactions may have an influence on the effective intercept of the two-particle correlation functions [22] and the effect has been shown to result in stronger drop in the intercept value for smaller source i.e. for hadronic strings created in lepton-lepton, lepton-hadron or hadron-hadron collisions. However, more detailed calculations indicate that the higher order corrections for

the final state interactions may very well cancel the first order effects resulting in a very small total final state interaction correction [23]. In our calculations these effects have been neglected, so our results should in principle be compared to data which present the *genuine* Bose-Einstein correlation function. This function is not necessarily the same as the short-range part of the two-particle correlation function since Coulomb and Yukawa interactions as well as the $\eta'\eta$ decay chain and other short-range correlations may mask the quantum statistical correlation functions. However, as the particle density is increased (in the limit of very large energy or large colliding nuclei), the Bose-Einstein correlations together with the final state interactions dominate over other short-range correlations due to combinatoric reasons.

IV. THE MODEL OF THE CORE

For central heavy ion collisions at high energies the beam or z axis becomes a symmetry axis. Since the initial state of the reaction is axially symmetric and the equations of motion do not break this pattern, the final state must be axially symmetric too. However, in order to generate the thermal length-scales in the transverse directions, the flow-field must be either three-dimensional, or the temperature must have significant gradients in the transverse directions. Furthermore, the local temperature may change during the the duration of the particle emission either because of the re-heating of the system caused by the hadronization and/or intensive rescattering processes or the local temperature may decrease because of the expansion and the emission of the most energetic particles from the interaction region.

We model the emission function of the core for high energy heavy ion reactions as as

$$S_c(x,K)\, d^4x = \frac{g}{(2\pi)^3}\, m_t \cosh(\eta - y)\, \exp\left(- \frac{K\cdot u(x)}{T(x)} + \frac{\mu(x)}{T(x)} \right) H(\tau) d\tau\, \tau_0 d\eta\, dr_x\, dr_y, \quad (21)$$

$$u(x) \simeq \left(\cosh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2} \right),\ b\frac{r_x}{\tau_0},\ b\frac{r_y}{\tau_0},\ \sinh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2} \right) \right), \quad (22)$$

$$T(x) = \frac{T_0}{\left(1 + a^2 \frac{r_x^2 + r_y^2}{2\tau_0^2}\right)\left(1 + d^2 \frac{(\tau - \tau_0)^2}{2\tau_0^2}\right)} \tag{23}$$

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - \frac{r_x^2 + r_y^2}{2R_G^2} - \frac{(\eta - y_0)^2}{2\Delta\eta^2}. \tag{24}$$

We include a finite duration, $H(\tau) \propto \exp(-(\tau - \tau_0)^2/(2\Delta\tau^2))$, $\tau = \sqrt{t^2 - z^2}$, τ_0 is the mean emission time, $\Delta\tau$ is the duration of the emission in (proper) time, $u(x)$ is the four-velocity of the expanding matter, $\mu(x)$ is the chemical potential and $T(x)$ is the local temperature characterizing the particle emission.

This emission function corresponds to a Boltzmann approximation to the local momentum distribution of a longitudinally expanding finite system which expands into the transverse directions with a non-relativistic transverse flow. The decrease of the temperature in the transverse direction is controlled by the parameter a, while the strength of the transverse flow is controlled by the parameter b. The parameter $c = 1$ is reserved to denote the speed of light, and the parameter d controls the strength of the change of the local temperature during the course of particle emission.

For the case of $a = b = d = 0$ we recover the case of longitudinally expanding finite systems. The finite geometrical and temporal length-scales are represented by the transverse geometrical size R_G, the geometrical width of the space-time rapidity distribution $\Delta\eta$ and by the mean duration of the particle emission $\Delta\tau$. We assume here that the finite geometrical and temporal scales as well as the transverse radius and proper-time dependence of the inverse of the local temperature can be represented by the mean and the variance of the respective variables i.e. we apply a Gaussian approximation, corresponding to the forms listed above, in order to get analytically trackable results. We have first proposed the $a = 0, b = 1$ and $d = 0$ version of the present model, and elaborated also the $a = b = d = 0$ model [12] corresponding to longitudinally expanding finite systems with a costant freeze-out temperature and no transverse flow. Soon the parameter b has been introduced [26] and

it has been realized that the transverse flow has to be non-relativistic at the saddle-piont corresponding to the maximum of the emission function. Yu. Sinyukov and collaborators classified the various classes of the ultra-relativistic transverse flows [24], [25], and introduced a parameter which controls the transverse temperature profile, corresponding to the $a \neq b = 0$ case. We have studied [21] the model-class $a, b, d = 0$ which we extend here to the $d \neq 0$ case too.

The integrals of the emission function are evaluated using the saddle-point method [24–26]. The saddle-point equations are solved in the LCMS [12], the longitudinally co-moving system, for $\eta_s << 1$ and $r_{x,s} << \tau_0$. These approximations are warranted if $|y - y_0| << 1 + \Delta\eta^2 m_t/T_0$ and $\beta_t = p_t/m_t << (a^2 + b^2)/b$. The flow is non-relativistic at the saddle-point if $\beta_t << (a^2 + b^2)/b^2$. The radius parameters are evaluated here up to $\mathcal{O}(r_{x,s}/\tau_0) + \mathcal{O}(\eta_s)$, keeping only the leading order terms in the LCMS. However, when evaluating the invariant momentum distribution (IMD), sub-leading terms coming from the $\cosh(\eta - y)$ pre-factor are also summed up, since this factor influences the IMD in the lower m_t region where data are accurate.

A. Bose-Einstein correlations

The experimental BECF is parameterized in the form of $C(Q_L, Q_{side}, Q_{out}) = 1 + \lambda \exp(-R_L^2 Q_L^2 - R_{side}^2 Q_{side}^2 - R_{out}^2 Q_{out}^2)$ where the intercept parameter λ and the radius parameters may depend on the rapidity and the transverse mass of the pair. The parameters of the the correlation function are given by

$$R_{side}^2 = R_*^2, \tag{25}$$

$$R_{out}^2 = R_*^2 + \beta_T^2 \Delta\tau_*^2, \tag{26}$$

$$R_L^2 = \tau_0^2 \Delta\eta_*^2 \tag{27}$$

We obtain [12,13]

$$\frac{1}{R_*^2} = \frac{1}{R_G^2} + \frac{1}{R_T^2}\cosh[\eta_s] \tag{28}$$

$$\frac{1}{\Delta\eta_*^2} = \frac{1}{\Delta\eta^2} + \frac{1}{\Delta\eta_T^2}\cosh[\eta_s] - \frac{1}{\cosh^2[\eta_s]}, \tag{29}$$

$$\frac{1}{\Delta\tau_*^2} = \frac{1}{\Delta\tau^2} + \frac{1}{\Delta\tau_T^2}\cosh^2[\eta_s]. \tag{30}$$

where the thermal length-scales are given by

$$R_T^2 = \frac{\tau_0^2}{a^2 + b^2}\frac{T_0}{m_t}, \tag{31}$$

$$\Delta\eta_T^2 = \frac{T_0}{m_t}, \tag{32}$$

$$\Delta\tau_T^2 = \frac{\tau_0^2}{d^2}\frac{T_0}{m_t}. \tag{33}$$

As a consequance, the parameters of the BECF-s are dominated by the smaller of the geometrical and the thermal scales not only in the spatial but in the temporal directions too. These analytic expressions indicate that the BECF views only a part of the space-time volume of the expanding systems, which implies that even a complete measurement of the parameters of the BECF as a function of the mean momentum K may not be sufficient to determine uniquely the underlying phase-space distribution. We also can see that for pairs with $\mid y_0 - Y \mid << 1 + \Delta\eta^2 M_t/T_0$ the terms arising from the non-vanishing values of η_s can be neglected. In this approximation, the cross-term generating hyperbolic mixing angle $\eta_s \approx 0$ thus we find the leading order LCMS result:

$$C(Q;K) = 1 + \lambda_*(K)\exp(-R_L^2 Q_L^2 - R_{side}^2 Q_{side}^2 - R_{out}^2 Q_{out}^2), \tag{34}$$

with a vanishing out-long cross-term, $R_{out,L} = 0$. $\lambda_*(K)$ is given by eqs. (14) or (16). According to the previous part, $\lambda_*(K) = f_c^2$ in case of the NA44 measurements.

The difference of the side and the out radius parameters is dominated by the lifetime-parameter $\Delta\tau_*$. Thus vanishing difference in between the R_{out}^2 and R_{side}^2 can be generated dynamically in the case when the duration of the particle emission is large, but the thermal duration $\Delta\tau_T$ becomes sufficiently small. This in turn can be associated with intensive changes in the local temperature distribution during the course of the particle emission. If the finite source sizes are large compared to the thermal length-scales we have

$$\Delta\tau_*^2 = \Delta\tau_T^2 \frac{1}{1 + \dfrac{\Delta\tau_T^2}{\Delta\tau^2}} \approx \frac{\tau_0^2}{d^2} \frac{T_0}{m_t} \tag{35}$$

$$R_L^2 \approx \tau_0^2 \frac{T_0}{m_t} \tag{36}$$

$$R_{side}^2 \approx \frac{\tau_0^2}{a^2 + b^2} \frac{T_0}{m_t} \tag{37}$$

Thus if $d^2 >> a^2 + b^2 \approx 1$ the model features a dynamically generated vanishing difference between the side and out radii.

If the vanishing duration parameter is generated dynamically, the model predicts an m_t - scaling for the life-time parameter as

$$\Delta\tau_*^2 = \frac{R_{out}^2 - R_{side}^2}{\beta_t^2} \simeq \frac{1}{m_t}, \tag{38}$$

Note that this prediction could be checked experimentally if the error bars of the measured radius parameters could be decreased to such a level that the difference between the out and the side radius parameters could be significant.

If the finite source sizes are large compared to the thermal length-scales and if we also have $a^2 + b^2 \approx 1$, one obtains an m_t *-scaling* for the parameters of the BECF,

$$R_{side}^2 \simeq R_{out}^2 \simeq R_L^2 \simeq \tau_0^2 \frac{T_0}{m_t}, \quad \text{valid for} \quad \beta_t << \frac{(a^2 + b^2)}{b^2} \simeq \frac{1}{b^2}. \tag{39}$$

Note that this relation is independent of the particle type and has been seen in the recent NA44 data [19]. This m_t-scaling may be valid to arbitrarily large transverse masses with $\beta_t \approx 1$ if $b^2 << 1$. Thus, to generate the vanishing difference between the side and out radius and the m_t scaling simultaneously, the parameters have to satisfy $b^2 << a^2 + b^2 \approx 1 << d^2$, i.e. the fastest process is the cooling, the next dominant process within this phenomenological picture has to be the development of the transverse temperature profile and finally the transverse flow shall be relatively weak.

B. Invariant momentum distributions

The IMD plays a *complementary role* to the measured Bose-Einstein correlation function [11–13]. Namely, the width of the rapidity distribution at a given m_t as well as T_* the effective temperature at a mid-rapidity y_0 shall be dominated by the *longer* of the thermal and geometrical length-scales. Thus a *simultaneous analysis* of the Bose-Einstein correlation function and the IMD may reveal information both on the temperature and flow profiles and on the geometrical sizes. E.g. the following relations hold:

$$\Delta y(m_t)^2 = \Delta\eta^2 + \Delta\eta_T^2(m_t), \qquad \text{and} \qquad \frac{1}{T_*} = \frac{f}{T_0 + T_G(m_t = m)} + \frac{1-f}{T_0}. \tag{40}$$

The geometrical contribution to the effective temperature is given by $T_G = T\, R_G^2/R_T^2$ and the fraction f is defined as $f = b^2/(a^2+b^2)$, satisfying $0 \le f \le 1$. The saddle-point sits at $\eta_s = (y_0 - y)/(1 + \Delta\eta^2/\Delta\eta_T^2)$, $r_{x,s} = \beta_t b R_*^2/(\tau_0 \Delta\eta_T^2)$, $r_{y,s} = 0$ and $\tau_s = \tau_0$.

For the considered model, the invariant momentum distribution of the core can be calculated as

$$\frac{d^2 n_c}{dy\, dm_t^2} = \frac{g}{(2\pi)^3} \exp\left(\frac{\mu_0}{T}\right) m_t \,(2\pi\Delta\eta_*^2\tau_0^2)^{1/2}\,(2\pi R_*^2)\frac{\Delta\tau_*}{\Delta\tau} \cosh(\eta_s)\, \exp(+\Delta\eta_*^2/2) \times$$
$$\times \exp\left(-\frac{(y-y_0)^2}{2(\Delta\eta^2 + \Delta\eta_T^2)}\right) \exp\left(-\frac{m_t}{T_0}\left(1 - f\,\frac{\beta_t^2}{2}\right)\right) \exp\left(-f\,\frac{m_t\beta_t^2}{2(T_0+T_G)}\right). \tag{41}$$

This IMD has a rich structure: it features both a rapidity-independent and a rapidity-dependent low-pt enhancement as well as a high-pt enhancement or decrease.

V. APPLICATIONS TO NA44 DATA

We have fitted the NA44 preliminary invariant momentum distributions for pions and kaons together with the final NA44 data [19] for the m_t dependence of the BECF parameters for both pions and kaons. As λ has been shown not to be dependent on m_t the IMDs measured are proportional to the IMDs of the core. Fixing the parameter $y_0 = 3$ we get a description of the IMD and the BECF for both pions and kaons. The result of the preliminary analysis, see Table, indicates large geometrical source sizes for the pions, $R_G(\pi) \approx 7$ fm and $R_L \approx 20$ fm, late freeze-out times of $\tau_0 \approx 7$ fm/c, a vanishing duration $\Delta\tau$ for both pions and kaons and finally surprisingly low freeze-out temperatures, $T_0 \approx 100$ MeV. The kaons appear from a smaller region of $R_G(K) \approx 4$ fm.

Let us mention, that the measured points for the IMD contain statistical errors only, systematic errors are not included. This implies that the errors of the parameters, as stated in the Table, are probably to be enlarged, when we can use final data. The $\Delta\chi^2$ is relatively large from the pion invariant momentum distribution. We have scanned the parts which contribute to the large increase in χ^2 and concluded that this increase seems to be related to some large fluctuations of data points at the edge of the acceptance region.

VI. CONCLUSIONS

In summary we have studied the case when the central boson-emitting region is surrounded by a large halo, which also emits bosons. If the size of the halo is so large that it cannot be resolved in Bose-Einstein correlation measurements, lot of information shall be concentrated in the momentum dependence of the intercept parameter of the correlation function. We have shown that with the help of the Bose-Einstein correlation measurement, the invariant momentum distribution can be measured for the two independent components

belonging to the core and the halo, respectively. The results do not depend on any particular parametrization of the core nor of the halo.

Analysis of the NA44 data for two-pion correlations indicated that the normalized invariant momentum distribution of the pions from the halo of long-lived resonances within errors coincides with the normalized invariant momentum distribution of the pions from the central core. This result can be explained with a hydrodynamical model of the core development in space-time. The number of pions coming from the halo region, ≥ 20 fm, was found to be 25 ± 2 % of the total number of pions within the NA44 acceptance.

Instead of observing a small fireball we are observing a big and expanding snow-flurry at CERN SPS $S + Pb$ reactions accordingly to this analysis of the partly preliminary NA44 data. The central temperature in the snow-flurry is slightly higher than in the outer regions, however the temperature gradient is rather small ($a^2 = 0.03 \pm 0.01$). The temporal changes of the local temperature during the particle emission seem to be rather intensive ($d^2 = 0.9 \pm 0.3$).

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Table

Results of a simultaneous fit to NA44 preliminary invariant momentum distributions and Bose-Einstein radius parameters for π and K in central $S + Pb$ reactions at 200 AGeV

χ^2/ndf, full fit	490/220
$\Delta\chi^2$, pion singles(128)	351
$\Delta\chi^2$, kaon singles(94)	137
$\Delta\chi^2$, radii measurements(9)	2
T_0 [MeV]	100±2
τ_0 [fm/c]	6.9±0.2
$R_{G(\pi)}$ [fm]	6.9±0.4
$R_{G,(K)}$ [fm]	4.1±0.4
$R_{G,L}$ [fm]	21±6
$\Delta\tau(\pi)$ [fm/c]	0.1±0.1
$\Delta\tau(K)$ [fm/c]	6±4
a^2	0.03±0.01
b^2	0.85±0.01
d^2	0.9±0.3

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